SPONTANEOUS INTRINSIC DECOHERENCE IN RABI OSCILLATIONS EXPERIMENTS

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Abstract We present a simple theoretical description of a recent QED experiment where damping of Rabi oscillations cannot be attributed to environment-induced decoherence (EID). We generalize a previous formalism to describe also non-exponential decay. The possibility of spontaneous intrinsic decoherence (SID), i.e., not due to interaction with environment or fluctuation of some internal parameter but of purely quantum mechanical origin, is discussed. Moreover an experimental cavity QED test of SID is suggested.

1. INTRODUCTION

Even though decoherence is a very general phenomenon [1], it is very difficult to verify it experimentally because most often the physical nature of the environmental degrees of freedom, responsible for the decoherence process, remains unknown. The only controlled experimental verification of decoherence has been given by the experiment of Ref. [2], in which the progressive transformation of a linear superposition of two coherent states of a microwave cavity mode into the corresponding statistical mixture has been monitored. In this case, the environmental decoherence has been checked with no fitting parameters because its physical origin, i.e., photon leakage out of the cavity, was easily recognizable and measurable. In this case, it is even possible to control decoherence, i.e., to considerably suppress its effects, for example by using appropriately designed feedback schemes [3].

In some cases however, the mechanisms responsible for decoherence are not easily individuated and examples are provided by a recent experiment which observed Rabi oscillations between two circular states of a Rydberg atom in a high-Q cavity [4]. In this case one observes damped oscillations to a steady state in which the population of each of the two levels approaches 1/2. A number of candidates have been already considered as possible physical sources of decoherence in this case: dark counts of the atomic detectors, dephasing collis
sions with background gas or stray magnetic fields within the cavity [4, 5] have been proposed as possible sources of decoherence. The only established fact is that, differently from Ref. [2], in this case, decoherence has a non-dissipative origin. In fact, the observed decay of the Rabi oscillations is much faster than the energy relaxation rate in this experimental configuration. Moreover, the fact that the population of each of the two levels asymptotically approaches 1/2 cannot be explained in terms of dissipative mechanisms as the photon leakage out of the cavity, which would take all the atoms in the ground state.

A quantitative explanation of the observed decay rate of the Rabi oscillations was just presented in Ref. [6], where we used a different approach to decoherence, proposed in Ref. [7], where a model-independent formalism has been derived to describe decoherence. The idea underlying the approach of Ref. [7] is the fact that the interaction time, i.e., the time interval in which the effective Hamiltonian evolution takes place, is a random variable. This randomness can have different origins depending on the studied system. In the case of the Rydberg atom experiment [4], the interaction time is determined by the transit time of the velocity-selected atom through the high-Q microwave cavity. This interaction time is random, due to fluctuations of the atomic velocities, which can have classical origin [6] or purely quantum mechanical origin as suggested in this paper. This randomness implies random phases $e^{-i E_n t' / \hbar}$ in the energy eigenstates basis. The experimental results unavoidably average over these random phases and this leads to decoherence, i.e., to the decay of off-diagonal matrix elements of the density operator in the energy basis. Therefore, as we shall see, our approach will give a generalized phase-destroying master equation, able to describe many situations in which decoherence is associated with random phases, originating for example from some frequency or interaction time fluctuations.

2. THE FORMALISM

First of all we recall the main points of the new formalism (see also Ref. [6, 7]). Let us consider an initial state described by the density operator $\rho(0)$ and consider the case of a random evolution time. The experimentally observed state is not described by the usual density matrix of the whole system $\rho(t)$, but by its time averaged counterpart [7]

$$\bar{\rho}(t) = \int_0^\infty dt' P(t, t', \tau) \rho(t') ,$$

where $\rho(t') = \exp\{-i Lt'\}\rho(0)$ is the usual unitarily evolved density operator from the initial state and $L \cdots = [H, \cdots] / \hbar$. Hence one can write

$$\bar{\rho}(t) = V(t) \rho(0) ,$$

(2)
where

\[ V(t) = \int_0^\infty dt' P(t, t', \tau) e^{-iLt'} \quad (3) \]

In Ref. [7], the function \( P(t, t', \tau) \) has been determined so to satisfy the following conditions: i) \( \tilde{\rho}(t) \) must be a density operator, i.e., it must be selfadjoint, positive-definite, and with unit-trace. This leads to the condition that \( P(t, t', \tau) \) must be non-negative and normalized, i.e., a probability density in \( t' \) so that Eq. (1) is a completely positive mapping. ii) \( V(t) \) satisfies the semigroup property \( V(t_1 + t_2) = V(t_1)V(t_2) \), with \( t_1, t_2 \geq 0 \). These requirements are satisfied by [7]

\[ V(t) = (1 + i\tau)^{-t/\tau} \quad (4) \]

\[ P(t, t', \tau) = \frac{e^{-t'/\tau}}{\tau} \left( \frac{t'/\tau}{\Gamma(t/\tau)} \right)^{(t/\tau)-1} \quad (5) \]

The above expressions \( V(t) \) and \( P(t, t', \tau) \) satisfy Eq. (3) according to the \( \Gamma \)-function integral identity [8, 9]. \( P(t, t', \tau) \) is the well known positive definite \( \Gamma \)-distribution function for the random variable \( t' \) and it parametrically depends on the clock time \( t \) and on the scaling time \( \tau \). The parameter \( \tau \) characterizes the strength of the evolution time fluctuations. When \( \tau \to 0 \), \( P(t, t', \tau) \to \delta(t-t') \) so that \( \tilde{\rho}(t) = \rho(t) \) and \( V(t) = \exp\{-iLt\} \) is the usual unitary evolution. However, for finite \( \tau \), the evolution operator \( V(t) \) of Eq. (4) describes decoherence in the energy representation (i.e., the approach to diagonal form), whereas the diagonal matrix elements remain constant, i.e., the energy is still a constant of motion \( \text{(non dissipative decoherence)} \). Furthermore by differentiating with respect to time Eq. (2) and using (4), one gets the following master equation for \( \tilde{\rho}(t) \)

\[ \dot{\tilde{\rho}}(t) = -\frac{1}{\tau} \log (1 + i\tau) \tilde{\rho}(t) \quad (6) \]

If one expands the logarithm at second order in \( \tau \), one obtains

\[ \dot{\tilde{\rho}}(t) = -\frac{i}{\hbar} [H, \tilde{\rho}(t)] - \frac{\tau}{2\hbar^2} [H, [H, \tilde{\rho}(t)]] \quad , (7) \]

which is the well-known phase-destroying master equation [10]. Hence Eq. (6) appears as a \textit{generalized} phase-destroying master equation taking into account higher order terms in \( \tau \). Notice, however, that the present approach is different from the usual master equation approach in the sense that no perturbative and specific statistical assumptions are made.

3. SID IN RABI OSCILLATIONS EXPERIMENTS

In a previous work [6] we applied this formalism to the two experiments of Refs. [4, 11]. Now we focus our attention on the one of Ref. [4], where the
resonant interaction between a quantized mode in a high-Q microwave cavity (with annihilation operator \( a \)) and two circular Rydberg states (\( |e\rangle \) and \( |g\rangle \)) of a Rb atom is studied. This interaction is well described by the usual Jaynes-Cummings model [12], which in the interaction picture reads

\[
H = \hbar \Omega_R \left( |e\rangle \langle g| a + |g\rangle \langle e| a^\dagger \right),
\]

where \( \Omega_R \) is the Rabi frequency. The Rabi oscillations, describing the exchange of excitations between atom and cavity mode, are studied by injecting the velocity-selected Rydberg atom, prepared in the excited state \( |e\rangle \), in the high-Q cavity and measuring the population of the lower atomic level \( g \), \( P_{eg}(t) \) as a function of the interaction time \( t \), which is varied by changing the Rydberg atom velocity. In the case of vacuum state induced Rabi oscillations, the decoherence effect is particularly evident and the Hamiltonian evolution according to Eq. (8) predicts

\[
P_{eg}(t) = \frac{1}{2} \left( 1 - \cos(2\Omega_R t) \right).
\]

Experimentally instead, damped oscillations are observed, which are fitted by

\[
P_{eg}^{\text{exp}}(t) = \frac{1}{2} \left( 1 - e^{-\gamma t} \cos(2\Omega_R t) \right),
\]

where the decay time fitting the experimental data is [5] \( \gamma^{-1} \approx 40 \mu \text{sec} \) and the corresponding Rabi frequency is \( \Omega_R/2\pi = 25 \text{ kHz} \). This decay of quantum coherence cannot be associated with photon leakage out of the cavity because the cavity relaxation time is larger (220 \( \mu \text{sec} \)) and also because in this case one would have an asymptotic limit \( P_{eg}^{\text{exp}}(\infty) = 1 \). The damped behavior of Eq. (10) is instead easily obtained if one applies the approach described above. In fact, from the linearity of Eq. (1), one has that the time averaging procedure is also valid for mean values and matrix elements of each subsystem. Therefore one has

\[
\tilde{P}_{eg}(t) = \int_0^\infty dt' P(t', t', r)P_{eg}(t') .
\]

Using Eqs. (2), (4), (5) and (9), Eq. (11) can be rewritten in the same form of Eq. (10):

\[
\tilde{P}_{eg}(t) = \frac{1}{2} \left[ 1 - \frac{\cos(\nu t)}{(1 + 4\Omega_R^2 r^2)^{1/2}} \right]
\]

\[
= \frac{1}{2} \left( 1 - e^{-\gamma t} \cos(\nu t) \right),
\]

where

\[
\gamma = \frac{1}{2\tau} \log \left( 1 + 4\Omega_R^2 r^2 \right)
\]
\[ \nu = \frac{1}{\tau} \arctan(2\Omega R T) \]  \hspace{1cm} (14)

We note that in general the time averaging procedure introduces not only a damping of the probability oscillations but also a frequency shift. However, if the characteristic time \( \tau \) is sufficiently small, i.e., \( \Omega R T \ll 1 \), there is no phase shift, \( \nu \approx 2\Omega R \), and

\[ \gamma = 2\Omega R T^2 \]  \hspace{1cm} (15)

The fact that in Ref. [4] the Rabi oscillation frequency essentially coincides with the theoretically expected one, suggests that the time \( \tau \) characterizing the fluctuations of the interaction time is sufficiently small so that it is reasonable to use Eq. (15). Using the above values for \( \gamma \) and \( \Omega R \), one can derive an estimate for \( \tau \), so to get \( \tau \approx 0.5 \mu \text{sec} \). This estimate is consistent with the assumption \( \Omega R T \ll 1 \) we have made, but, more importantly, it turns out to be comparable to the experimental value of the uncertainty in the interaction time. In fact, the fluctuations of the interaction time [4] \( \hat{t} = \sqrt{\sigma_0^2} \) (\( w \) is the cavity mode waist) are mainly due to the experimental uncertainty of the atomic velocity \( v \). Since \( w = 0.6 \text{ cm} \), the mean velocity is \( \bar{v} \approx 300 \text{ m/sec} \) and the velocity uncertainty is \( \sigma_v / \bar{v} = 1\% \) (see Ref. [4]), one has \( \hat{t} = \sqrt{\sigma_0^2} \approx 50 \mu \text{sec} \) and \( \tau \approx \hat{t} = \bar{v} / \sqrt{\sigma_0^2} = 0.5 \mu \text{sec} \), which is just the estimate we have derived from the experimental values. This simple and approximate argument supports the interpretation that the decoherence observed in Ref. [4] is essentially due to the randomness of the interaction time.

In reality the above argument is just a rough estimate of \( \tau \). A more precise evaluation requires a generalization of the previous theory. In fact, at the beginning of this paper we exposed our formalism with a constant \( \tau \). However we notice that complete positive mapping, described by Eqs. (1)–(5) and Eqs. (11)–(14), can be maintained also when \( \tau \) is a function of \( \hat{t} \). In such a case the semigroup property is dropped.

As a matter of fact in the experiment of Ref. [4] each experimental point at a given time \( \hat{t} \) is a different experiment, i.e., one selects with a proper laser excitation a velocity group with mean velocity \( \bar{v} \) such that

\[ \hat{t} = \sqrt{\pi} \frac{\sigma_v}{\bar{v}} \]  \hspace{1cm} (16)

sending one particle at a time across the cavity. In this way different values of \( \tau \) are obtained selecting different mean velocity group \( \bar{v} \). The spread of each velocity group, \( \Delta_v \), depends on the laser excitation. Hence the experimental uncertainty in interaction time \( \hat{t} \) can be written as

\[ \tau \approx a \hat{t} \]  \hspace{1cm} (17)

where we defined

\[ a \equiv \frac{\Delta_v}{\bar{v}} \]  \hspace{1cm} (18)
In the experimental situation of Ref. [4] $a \approx 0.01$ for all velocity group.

The above estimate of $\tau = 0.5 \mu$sec corresponds to replace $t$ in Eq. (17) a sort of mean interaction time of 50 $\mu$sec. Furthermore one should not forget that beside this classical velocity spread, $\Delta v$, there is an intrinsic velocity spread, $\sigma_v$, due to Heisenberg uncertainty principle (HUP)

$$\sigma_v = \frac{h}{2m}$$

(19)

where $m$ is the atomic mass and we have assumed that each atom can be described by a minimum uncertainty wave packet with initial position uncertainty $\sigma_0$. The total position spread at time $t$ can be written as

$$\Delta x(t) = \sqrt{\sigma_0^2 + \sigma^2 t^2 + \Delta^2 v t^2}$$

(20)

Here, the first two terms describes the particle wave packet Schrödinger spread and the last term is due to the classical velocity spread described above.

Eq. (20) can be justified by first principle as follows. Let us assume that atomic translational degree of freedom is represented by the following statistical operator

$$\rho(t) = \int_{-\infty}^{\infty} dv \mathcal{P}(v) \langle v, t \rangle \langle v, t \rangle$$

(21)

where $|v, t\rangle$ is a free particle gaussian wave packet with average velocity $v$, which corresponds to the classical velocity, i.e.,

$$|\langle x|v, t\rangle|^2 = |\psi(x, t)|^2 = \frac{1}{\sqrt{2\pi} \sigma_t} \exp \left\{ -\frac{(x - vt)^2}{2\sigma_t^2} \right\}$$

(22)

where $\sigma_t^2 = \sigma^2_0 + \sigma^2 v t^2$ is the well known Schrödinger free particle spread. $\mathcal{P}(v)$ represents the classical velocity distribution, which we assume to be gaussian:

$$\mathcal{P}(v) = \frac{1}{\sqrt{2\pi} \Delta_v} \exp \left\{ -\frac{(v - \bar{v})^2}{2\Delta_v^2} \right\}$$

(23)

Hence, with obvious notation,

$$P(x, t) \equiv \langle x|\rho(t)|x \rangle = \int_{-\infty}^{\infty} dv \mathcal{P}(v) |\langle x|v, t\rangle|^2$$

(24)

Using Eq. (22) and (23) and performing the integral (24), we obtain

$$P(x, t) = \frac{1}{\sqrt{2\pi} \Delta_x(t)} \exp \left\{ -\frac{(x - \bar{v}t)^2}{2\Delta_x(t)^2} \right\}$$

(25)

where $\Delta_x(t)$ is the same of Eq. (20).
We propose to take as value of \( \tau \) the uncertainty of interaction time, so that

\[
\tau \equiv \tau(t) = \frac{\Delta_n(t)}{\nu}
\]

(26)

where \( \tau \) and \( \nu \) are related by Eq. (16). Eq. (26) corresponds to the uncertainty of the arrival time [13] of the atoms at the end of the cavity, i.e., at the time \( t \) given by Eq. (16). For long enough times, the initial spread \( \sigma_0 \) can be neglected in Eq. (20), so that one can write \( \tau \approx at \) (see Eq. (17)), where

\[
a = \frac{\sqrt{\sigma_n^2 + \Delta_n^2}}{\nu}
\]

(27)

Let us notice that if the classical contribution is dominating, as it is in Ref. [4], Eqs. (26) and (27) reduce to Eqs. (17) and (18). If \( \sigma_n \) dominates \( \Delta_n \), then it is possible to see SID.

In general Eqs. (13) and (14) become

\[
\gamma(t) = \frac{1}{2at} \log \left( 1 + 4\Omega_R^2 \nu^2 t^2 \right)
\]

(28)

\[
\nu(t) = \frac{1}{at} \arctan \left( 2\Omega_R \nu t \right)
\]

(29)

where \( a \) is given by (27).

For short enough times, such that one has \( 2\Omega_R \nu t \ll 1 \), one obtains \( \nu \approx \Omega_R \) and \( \gamma \approx 2\Omega_R^2 \nu t \) so that damping becomes gaussian (i.e., non exponential) and

\[
\hat{P}_{eg}(t) = \frac{1}{2} \left( 1 - e^{-2\Omega_R^2 \nu t^2} \cos \left( 2\Omega_R t \right) \right)
\]

(30)

For long enough times (i.e., short enough velocity group), this last approximation is no more valid and one has a power law decay given by Eq. (12) with \( \tau = at \).

Due to uncertainty of experimental points of Ref. [4], especially for the long time behaviour, it is difficult to decide which one is better between the exponential fit, described before and in Ref. [6], or the non-exponential one.

However, from Eq. (26) one sees that \( \tau \) can never be taken arbitrarily small, since [13] \( \tau \approx \sigma_0 / \nu \approx \hbar / 2\sigma(K) \), where \( \sigma(K) \) is the standard deviation of the kinetic energy \( K = p^2 / 2m \) of the atomic center of mass. However, since the total Hamiltonian of the system is the sum \( \mathcal{H} = K + H \), we have \( \sigma(\mathcal{H}) \geq \sigma(K) \) so that \( \tau \geq \hbar / \sigma(\mathcal{H}) \). This result is in agreement with the suggestion of Ref. [7]. In any case, even if the classical dispersion of velocity group is arbitrarily small, the limit \( \tau \to 0 \) cannot be taken, in fact Eq. (26) says that there is a finite, intrinsic value of \( \tau \) due to Schrödinger spread, whose minimum value is

\[
\tau_{\text{min}} = \sqrt{\frac{\hbar}{m \nu^2}}
\]

(31)
obtained for $\sigma_0 = \sqrt{\hbar /2m}$. When the classical contribution in Eq. (20) is negligible, one should observe spontaneous intrinsic decoherence (SID), i.e. decoherence which is not induced by environment or by fluctuations of some experimental parameters (spontaneous) but of purely quantum mechanical origin (intrinsic). Hence SID should be observable at all times in the experimental apparatus of Ref. [4] if the classical velocity spread $\Delta_v$ is reduced and it becomes comparable or smaller than the quantum one, $\sigma(v)$, in Eq. (20).

In conclusion, referring to a specific experimental situation where environment induced decoherence is negligible, we have explicitly shown that there is another source of decoherence due to fluctuation of the interaction time. These fluctuations can be originated by classical velocity spread or by intrinsic quantum velocity spread, due to the Heisenberg uncertainty principle: when this is dominant, SID should be observed.

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References

[9] In Ref. [7] a more general expression for $P(t, t')$ and $V(t)$, depending on two parameters $\tau_1$ and $\tau_2$ is derived. We choose $\tau_1 = \tau_2 = \tau$ because in the experiments considered here, the effective interaction time $\langle t' \rangle = t\tau_1 / \tau_2$ [7] has to coincide with the "laboratory time" $t$, implying therefore $\tau_1 = \tau_2$.